Chinese Journal of Electronics
Vol.10, No.2, Apr. 2001

Exploiting Same Scale Similarity in Fisher's Scheme*

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Abstract — The method proposed by Y. Fisher is the most popular fractal image coding scheme. In his scheme, domain blocks are constrained to be twice as large as range blocks in order to ensure the convergence of the iterative decoding stage. However, this constraint has limited the fractal encoder to exploit the self-similarity of the original image. In order to overcome the shortcoming, a novel scheme using same-sized range and domain blocks is proposed in the paper. Experimental results show the improvements in compression performance.

Key words — Fractal image coding, Image coding, Digital image compression.

I. Introduction

Fractal image coding was proposed in about 1988[1] and since then it has aroused much attention for its novel idea[2−5]: It uses the concept of contractive transform and the fixed point theorem. The first automatic scheme which can compress monochromatic images was proposed by A.E. Jacqin[2]. After that, Y. Fisher modified the partition strategy and proposed a more practical scheme which can achieve better performance and has become the most common scheme[3]. In both schemes, the original image is first partitioned into range and domain blocks and they constrained that domain blocks must be twice larger than range blocks. In fact, in traditional fractal coding scheme, range blocks do not find the similar part in the original image, instead, they try to find the most similar parts in the down-sampled image. The coding diagram is shown in Fig.1. The constraint on the partition ensures the convergence of the decoding. However, it brings some shortcomings: First, the constraint does not exploit the self-similarity of the same scale which is most common in natural images; Second, it affects the amount of domain blocks and will decrease the similarity degree between a range block and its matched domain block.

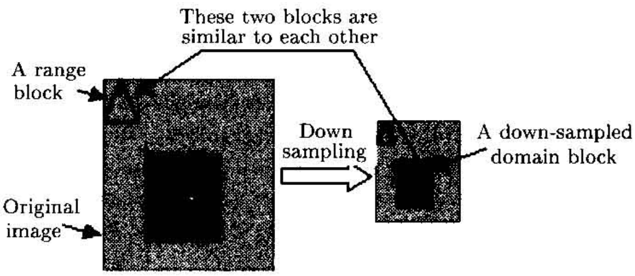

Fig. 1. The coding procedure of traditional fractal image coding

From the above analysis, we know that if we try to use the self-similarity of the same scale, the coding performance must be improved. However, same-sized domains can not be used directly, otherwise, the convergence could be a big problem. Next, we will propose a practical scheme which uses domain blocks of same size as ranges.

II. Our Scheme

Using same-sized domain blocks in the coding of rang blocks is an efficient way to exploit the same scale self-similarity of the original image. However, the convergence condition should be considered at the same time, and some measures must be carried out for the purpose. The concise coding steps are as follows:

(*a*) For an original image $X_{\rm orig}$ to be encoded, create a flag image $X_{\rm flag}$ which is the same-sized as the original. It is used to make some flags during the encoding, and every pixel of the flag image is initiated as zero;

(*b*) Quadtree partitions the original image into range blocks and domain blocks like Fisher's. Unlike Fisher's, the domain blocks pool $\mathcal{D}$ includes two kinds: $\mathcal{D}_1$ and $\mathcal{D}_2$. $\mathcal{D}_1$ consists of the same-sized ones, while $\mathcal{D}_2$ consists of ones twice the range size, that is, $\mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_2$;

*Manuscript Received Apr. 2000; Accepted Aug. 2000. This work is jointly supported by the National Natural Science Foundation of China (No.69802001), Chinese Ministry of Education and National Key Lab. of Pattern Recognition.

(c) For every range block to be encoded, check every pixel in the same area of X_{flag}. If all pixels are zero, it means that we can search $\mathcal{D}_1$ or $\mathcal{D}_2$ for the best matching domain. If not, we can only search $\mathcal{D}_2$;

(d) If a range block is encoded with $\mathcal{D}_1$, just calculate the rms(root mean square) error between the range and a domain in $\mathcal{D}_1$. If the error is lower than a pre-selected threshold, save the coordinates of the domain, meanwhile change the pixel value of the same domain area of X_{flag} to 255. If all the domains in $\mathcal{D}_1$ can not satisfy the threshold, search $\mathcal{D}_2$. The coding method for twice domain blocks is the same as Fisher's. If all the twice domain blocks can not satisfy the threshold, subdivide the range block into four quadrants and go to step (c);

(e) Repeat the step (c) to (d) until every range is encoded.

III. Contractivity of Our Method

If a range block is encoded with a larger domain block, the transform involved must be contractive. We now just consider the transform contractivity of range blocks encoded with same-sized domains.

For a range block encoded with a same-sized domain, the encoding procedure can be shown in Fig.2. The range block is first encoded with a same-sized domain, and the transform involved is denoted as T_1. From step (c), we know that any part of the domain area must be encoded with a twice larger domain, and the transform involved is denoted as T_2.

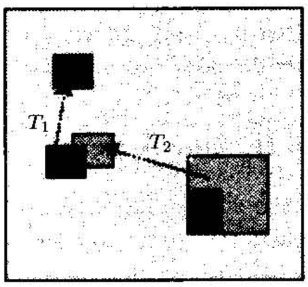

Fig. 2. The coding procedure of a range block with same-sized domain

In fractal image coding, every transform for a range block can be written as an affine transform.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} e \\ f \\ o \end{bmatrix} \tag{1}$$

where (x, y) is the pixel position, z is the pixel value in (x, y), $(X, Y)$is the transformed pixel position and Z is the transformed pixel value. a, b, c, d, e, f, α and o are the parameters of the transform.

For a range block encoded with a same-sized domain block, the affine transform T_1 can be written as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} e_1 \\ f_1 \\ 0 \end{bmatrix} \tag{2}$$

From step (c), we know that the pixel (x_1, y_1, z_1) must be transformed by a contractive transform T_2:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} a_2 & b_2 & 0 \\ c_2 & d_2 & 0 \\ 0 & 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} + \begin{bmatrix} e_2 \\ f_2 \\ o_2 \end{bmatrix} \tag{3}$$

From Eqs.(2) and (3), we get:

$$\begin{aligned} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} a_2 & b_2 & 0 \\ c_2 & d_2 & 0 \\ 0 & 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} + \begin{bmatrix} e_2 \\ f_2 \\ o_2 \end{bmatrix} \right) + \begin{bmatrix} e_1 \\ f_1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} a_2 & b_2 & 0 \\ c_2 & d_2 & 0 \\ 0 & 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} + \begin{bmatrix} e_1 + e_2 \\ f_1 + f_2 \\ o_2 \end{bmatrix} \end{aligned} \tag{4}$$

That is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} a_2 & b_2 & 0 \\ c_2 & d_2 & 0 \\ 0 & 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} + \begin{bmatrix} e_1 + e_2 \\ f_1 + f_2 \\ o_2 \end{bmatrix} \tag{5}$$

The above transform is obviously a contractive transform.

IV. Experimental Results

Two experiments will be presented here. The test images used are $256 \times 256 \times 8$ "Xh" and $256 \times 256 \times 8$ "Girl" shown in Fig.3(a) and Fig.3(b).

Fig. 3. (a) The original $256 \times 256 \times 8$ test image "Xh"; (b) The original $256 \times 256 \times 8$ test image "Girl"

Experiment 1

This experiment is used to test the convergence of the proposed scheme. In step (c), we check the flag image X_{flag} to decide the use of the same-sized domain pool $\mathcal{D}_1$. In order to verify the efficiency of step (c), we encode a standard test image "Xh" with and without the check respectively. Fig.4(a) and Fig.4(b) are the flag images after encoding with and without the check, and Fig.4(c) and Fig.4(d) are the restored images with and without the check respectively. In the flag images, the white part

Fig. 4. (a) The flag image with the check of step (c); (b) The flag image without the check of step (c); (c) The restored image with the check of step (c); (d) The restored image without the check of step (c)

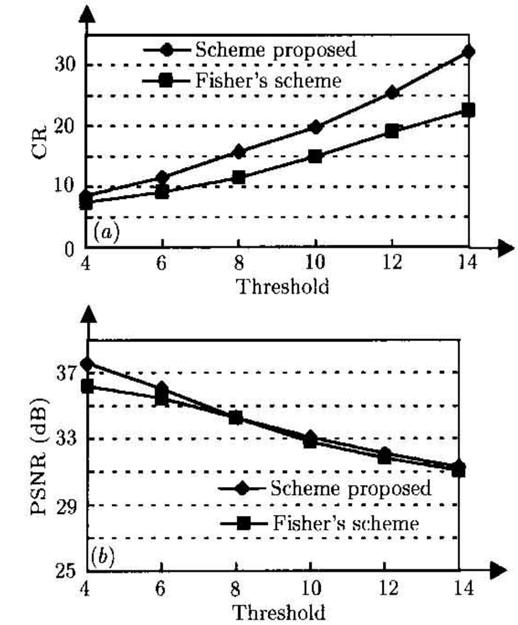

Fig. 5. (a) compression ratio (CR) comparison via different thresholds; (b) the peak-to-peak signal to noise ratio (PSNR) comparison via different thresholds

means that the area is encoded with the same-sized domains. Comparing Fig.4(a) and Fig.4(b), we know that the white area in Fig.4(b) is larger, resulting in higher compression of the encoding without the check. However, the restored image without the check can not converge to a perfect image in some parts shown in Fig.4(d).

Experiment 2

This experiment is done to compare the performance with the most common scheme– Fisher's quadtree partition scheme. The original image used is $256 \times 256 \times 8$ "Girl". The compression ratio (CR) comparison via different thresholds is shown in Fig.5(a), and the peak-to-peak signal to noise ratio (PSNR) comparison via different thresholds is shown in Fig.5(b).

From the comparison, we can easily conclude that the proposal can improve the performance remarkably.

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